Edge Detection in Gated Cardiac Nuclear Medicine Images

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Abstract
Mean Field Annealing using a piecewise linear model was applied to gated cardiac nuclear medicine images as a preprocessing tool for image smoothing and noise reduction. A second derivative operator was then used to extract the edges for ventricle boundary estimation. Combined with the user input initial boundary estimate, the extracted edge information was used to find a minimum cost boundary, which was optimum with regards to boundary smoothness and the boundary edge strength.

1. Introduction
Gated cardiac nuclear medicine imaging uses an ECG trigger to gate the acquisition of images during the cardiac cycle. Multiple slices are usually obtained and they can be played back in a cine loop. This allows visualization of the wall motion of the ventricle as well as the calculation of the ejection fraction, which provides a very important evaluation of the ventricle function.

The edge detection software developed at ADAC Corporation for gated cardiac nuclear medicine images implements a radial search algorithm that assigns different threshold values for edges in different directions on the acquired images smoothed by a 5 x 5 spatial filter. These threshold values are stored in a memory table and they remain constant for all the slices under study. Due to variations of brightness, contrast, and signal-to-noise ratio among different slices, this method may not be the most accurate method in extracting the left ventricle boundary. In this paper we propose an alternative approach using Mean Field Annealing as a preprocessing tool for image smoothing and noise removal. The Laplacian operator is then used to extract edges from the MFA smoothed images. Combined with the user input of initial boundary estimate, the extracted edge information is used in a minimum cost path search. The final boundary estimate is optimum with regards to boundary smoothness as well as to edge strength.

2. Method
A Markov Random Field based image processing technique, Mean field annealing (MFA), was used as a preprocessing tool for noise removal and image smoothing on the gated cardiac nuclear medicine images. A set of points plotted by a clinician served as an initial boundary estimate. A second order operators was applied to the MFA preprocessed image data to get edge information. Both the initial boundary estimate and the edge detection information were then fed to a dynamic programming optimization algorithm, which minimized a cost function and found the final boundary estimate.
2.1 MFA Smoothing

MFA is a minimization strategy which is a deterministic approximation to Simulated Annealing. MFA has been applied to restoration of locally-homogeneous and locally smooth range images [1],[2].

From the noise-corrupted measured image \( g \), we want to estimate the unknown ideal image \( f \). MFA restores the noisy image by maximizing the a-posteriori probability \( p(f | g) \). Using Bayes' rule, the a-posteriori probability can be expressed as the following proportionality:

\[
p(f | g) = p(g | f) p(f)
\]

(1)

We model the noise as additive, independent, and stationary zero-mean Gaussian. Thus the noise term \( p(g | f) \) is of the form:

\[
p(g | f) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\sum (g_i - f_i)^2}{2\sigma^2}\right)
\]

(2)

where the index \( i \) ranges all the pixels in the image. The noise term represents how the measured image resembles the 'ideal' image.

The prior term, which represents our knowledge of the property of the ideal image, depends on \( f \) only, as is represented in the following form,

\[
p(f) = \exp\left(-\frac{\sum V_i}{T}\right)
\]

(3)

where \( V \) is a function of the neighborhood pixels of pixel \( i \).

By taking the natural logarithm of both the left and the right side of Equation (1) and changing the sign, the objective function is defined as:

\[
H(f, g) = H_n(f, g) + H_p(f)
\]

(4)

where \( H_n \) and \( H_p \) are the noise and prior terms respectively, and are chosen as follows

\[
H_n(f) = \sum_i \frac{(f_i - g_i)^2}{2\sigma^2}
\]

(5)

\[
H_p(f) = \sum_i \left(-\frac{b}{2\pi T}\right) \exp\left(-\frac{\Lambda_i^2}{2T}ight)
\]

(6)

where \( \Lambda_i \) is some measure of the brightness change in the vicinity of pixel \( i \).

The restoration is performed to minimize \( H(f, g) \) because of the sign change. Minimizing \( H_n \) emphasizes that the restored image should resemble the measured image. On the other hand, minimization of \( H_p \) results in a restored image which satisfies our prior knowledge of the property of the ideal image.

In the gated cardiac nuclear medicine image, ventricle-ventricle and ventricle-atrium boundaries are roof edges. By choosing a linear-piecewise prior model, the roof edges of ventricle-ventricle and ventricle-atrium boundary are preserved. A second derivative operator, such as a
Laplacian, is then able to extract these edges. In this project, a piecewise-linear model using quadratic variation was chosen as follows:

\[ \Delta^2 = \left( \frac{\partial^2}{\partial x^2} \right)^2_i + \left( \frac{\partial^2}{\partial y^2} \right)_i + 2 \left( \frac{\partial^2}{\partial x \partial y} \right)_i \]  

(7)

This quadratic variation operator is implemented by convolving three kernels \( (h_{xx}, h_{yy}, h_{xy}) \) with the pixels in the 3 by 3 neighborhood of pixels \( i \), where

\[
\begin{align*}
  h_{xx} &= \frac{1}{6} \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\
  h_{yy} &= \frac{1}{6} \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix} \\
  h_{xy} &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\end{align*}
\]  

(8)

and

\[
H(f, s) = \left[ \frac{\sum (f_i - s_i)^2}{2\sigma^2} \right] - \beta \left[ \sum \frac{1}{\gamma \exp \left( -\frac{\langle f \otimes h \rangle_i^2}{2\gamma^2} \right)} \right]
\]  

(9)

where \( h \) is one of the three kernels listed above.

The annealing was carried out iteratively by taking the gradient descent of (9), which results in the following:

\[
f_{k+1} = f_k - \gamma \frac{\partial}{\partial f_i} H(f, s)
\]  

(10)

and the resulting derivative works out to be

\[
\frac{\partial}{\partial f_i} H = \left( \frac{f_i - s_i}{\sigma^2} + \frac{\beta}{\gamma \beta} \left( f \otimes h \right)_i \exp \left( -\frac{\langle f \otimes h \rangle_i^2}{2\gamma^2} \right) \otimes h_{rev} \right)
\]  

(11)

where \( \gamma \) is the relaxation ratio, and \( h_{rev} \) is the reverse kernel of the convolution kernel (8).

2.2 Boundary Detection

From the user input of the initial boundary estimate, a periodic, piecewise cubic spline was computed to interpolate these data points to form an initial boundary. At each user input point, the second derivative operator was applied to the pixels orthogonal to the initial estimated boundary on both sides. The number of pixels searched along the orthogonal line is variable based on the user selection. The obtained second derivatives generate an extracted data matrix, where the second derivatives along each orthogonal searching path form a column of this matrix, and the total number of columns is equal to the total number of the user input points.

The extracted data matrix, which contained both the user input estimate and the edge detection information was then used for the final boundary estimation.

To estimate a closed smooth boundary which tends to pass through the strongest edge points, it is proposed to minimize the following cost function\([3],[4]\):

\[
C(x) = (1 - \alpha) \sum_i (x_i - x_{i+1})^2 + \alpha \sum_i \left[ \max E_i - E_i(x_i) \right]
\]  

(12)

where \( \alpha \) is the weighting factor, \( x_i \) indicates the row number of the boundary path in the column.
i. If two neighboring points on the boundary pass through the same row \( k \), that is \( x_i = x_{i+1} = k \), then the first penalty term of (12) will be minimum for those two points. \( E_i(x_i) \) (second derivative) is the edge strength of the boundary point of column \( i \), which is at the column \( i \) and row \( x_i \). Max\( E_i \) is the maximum edge strength of the column \( i \).

The weighting factor \( \alpha \) can vary from 0 to 1. When \( \alpha \) is set to 0, the smoothest path is found without using the edge information. On the other hand, \( \alpha = 1 \) will result in the boundary passing through the strongest edge points with no penalty for sharp direction changes. For \( \alpha \) between 0 and 1, the final boundary estimate will be a compromise between the boundary smoothness constraint and the edge strength.

3. Result and Discussion

The MFA smoothing results were compared with the median filter and the 3 x 3 spatial filter. Figure 1(a) is the original blood pool image of the ventricle. The MFA restored image is shown in Figure 1(b). Both the median filter and the 3 x 3 blurring filter were applied to the original image for smoothing. The results are shown in Figure 1(c) and 1(d) respectively.

Figure 1: (a) is the original noisy image. (b) shows the MFA smoothed image. (c) Median filtered original image. (d) 3 by 3 averaging kernel applied on the original image.

Simple smoothing operators such as the median filter operate on pixels within a local 3 x 3 or 5 x 5 neighborhood and the same amount of smoothing is performed throughout the whole image. As a result, the small features in a low variance area are blurred away and only the big
features remain. MFA smoothing, on the other hand, preserves local features while removing random noise. The energy function (9) was minimized via MFA. We observe from Figure 1(b) that use of the $H_n$ of Equation (2) has preserved the sharp noise-generated peaks as well as the roof edges. This is due to the assumption underlying Equation (2); the noise is Gaussian. Since the noise in a nuclear medicine image is Poisson, a better noise model would be

$$p(g_i|f_i) = \frac{(\lambda g_i)^{g_i} \exp(-\lambda g_i)}{g_i!}$$

However, Han & Snyder has shown that equivalent results can be obtained by using local adaptive noise variance and local initial temperature in the annealing algorithm [5].

![Image of two contour plots](image)

**Figure 2:** (a) Cubic spline contour based on the initial user input. (b) The final boundary estimate vs. the manual drawn contour.

Figure 2(a) shows a cubic spline contour based on the initial user input estimate. The short lines which cross the boundary are the radial search lines. The second derivatives were calculated along these radial lines and they were used in the final estimate of the boundary. Figure 2(b) is the final estimated boundary. In comparison, the manually drawn contour (dotted line) is also shown in Figure 2(b). Our result is very close to the contour drawn manually by a radiologist. However, the semi-automatic generation of contours will be much more efficient than doing so manually. For a multiple slice study, the boundary estimate from the first slice can be used to estimate the boundary in the following slices with no or minimum human intervention, which greatly reduces the heavy load of manual generation of contours on each slice.

**Conclusion**

The current implementation of the system utilizes a Gaussian noise model, and consequently does not find step edges with sufficient reliability to run fully automatically. Incorporating a more accurate Poisson model is currently under progress.
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References


